

Theoretical Overview of the Pentaquark Baryons

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A review is given of the theoretical ideas concerning the mysterious pentaquark baryons proposed during the first year after its discovery. We focus on the difficulties involved with the constituent quark models and the discrepancy between the QCD predictions and most of the phenomenological models.

§1. Introduction

The year 2003 will be remembered as a renaissance of hadron spectroscopy. Indeed, there were many reports of the discovery of new narrow resonances. The list includes the celebrated pentaquark baryons,¹⁾ the new charmonium state $X(3872)$,²⁾ new D_s mesons,³⁾ and even a narrow K bound nucleus.⁴⁾ Among these the discovery of the first pentaquark baryon, Θ^+ , was the most striking event.

The exotic baryon Θ^+ was discovered in a seminal experiment performed by the LEPS group at the SPring-8 facility in Harima, Japan.¹⁾ This particle was identified in the K^+n invariant mass spectrum in the reaction $\gamma n \rightarrow K^- + \Theta^+ \rightarrow K^- + K^+ + n$, which was induced by a SPring-8 tagged photon beam of energy up to 2.4 GeV. That search was motivated by a theoretical paper by Diakonov, Petrov and Polyakov (DPP),⁵⁾ in which they predicted a narrow $S = +1$ baryon.

The reported mass of Θ^+ is 1540 ± 10 MeV, and the upper limit of its width is 25 MeV. As this baryon has strangeness +1, it should contain at least one \bar{s} , and thus the minimal number of quarks for Θ^+ is five. From the charge and the strangeness, $u^2 d^2 \bar{s}$ is a possibility as the content of Θ^+ . It is therefore commonly called the ‘pentaquark’.

The discovery of Θ^+ inspired many other experiments, and the existence of Θ^+ was soon confirmed by various groups, including ITEP (DIANA),⁶⁾ JLAB (CLAS)⁷⁾ and ELSA (SAPHIR).⁸⁾ Its discovery was followed by the discovery of yet another exotic baryon, Ξ^{--} , found by the NA49 group at CERN.⁹⁾ The particle $\Xi^{--}(s^2 d^2 \bar{u})$ is another manifestly exotic baryon, whose decay into $\Xi^- \pi^-$ has been observed at the mass $M = 1.862$ GeV with a width $\Gamma < 18$ MeV.

Since the discovery of Θ^+ was reported, numerous papers attempting to understand its properties and structure have been uploaded to the preprint server. In this article, we pick out some of the prominent achievements among them, attempt to explain their content, and also point out some problems. We concentrate on Θ^+ , first summarizing the experimental facts (§2) and then giving an overview of several important theoretical ideas (§§3 and 4). In particular, we discuss the predictions of the quark models with regard to the pentaquark states.

As QCD is the fundamental theory of hadrons, our goal is to understand Θ^+ from the viewpoint of QCD. Several attempts have been made using the QCD sum rule and lattice QCD simulations. In §5, we give an overview of the results and their interpretation. We give a conclusion in §6.

§2. Facts

A series of experiments have confirmed that the mass of Θ^+ is $M_{\Theta^+} \sim 1540$ MeV, while the upper limit of the width is 9 MeV, although the accumulated data exhibits some dispersion of the central value of the mass.¹⁰⁾ There is no consensus regarding the width, although the HERMES group¹¹⁾ claims a non-upper-limit width of order 10 MeV. (Their mass value is somewhat smaller, 1528 MeV.) Because it decays into nK^+ by the strong interaction, the conservation rules guarantee that it has a strangeness $S = +1$, baryon number $B = +1$, and charge $Q = +1$. Thus, the hypercharge is $Y \equiv B + S = 2$, and the third component of the isospin is $I_3 = 0$. No corresponding pK^+ ($I_3 = +1$) state is observed at the same mass. This leads to the conclusion that the isospin of Θ^+ is $I = 0$ (see Table I). It is obvious that this state cannot be composed of only three quarks. It also seems important that no $S = +1$ baryon state has been observed below the NK threshold, and thus this state seems to be the ground state.

As the flavor $SU(3)$ is an approximate symmetry of QCD, it is likely that Θ^+ belongs to an $SU(3)$ multiplet. The simplest $SU(3)$ irreducible representation (IR) for Θ^+ is $\overline{10}$, which is, in fact, unique if we restrict ourselves to the representations produced by 5 quarks (Fig. 1). This is the representation obtained from the Skyrmion model, and also it is that assumed in most of the quark model descriptions. We therefore assume here that Θ^+ is a member of $\overline{10}$.

I	I_3	B	S	Y	Q	J^π
0	0	1	1	2	1	$1/2^+, 1/2^-, \dots$

Table I. The quantum numbers of Θ^+ . The spin and parity have not been determined.

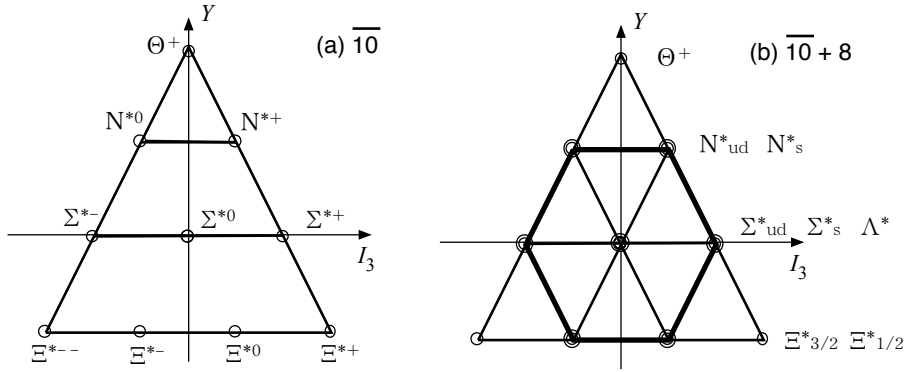


Fig. 1. (a) The $\overline{10}$ pentaquarks and (b) the mixing of the $\overline{10}$ plus 8 representations for the pentaquarks.

Where are the other members associated with $\overline{10}$? An important member is the penta-quark N^* ($I = 1/2$, $Y = 1$), which is the state with the flavor content:

$$N_{\overline{10}}^* = [(2/3)\bar{s}s + (1/3)(\bar{u}u + \bar{d}d)] \times (uud \text{ or } ddu). \quad (2.1)$$

Because this is a nucleon resonance, it is quite plausible that the present PDG table¹²⁾ already includes this state. DPP⁵⁾ boldly identified a $1/2^+$ nucleon resonance, $N(1710)$, as one such possibility. It is, however, noted that the $SU(3)$ breaking can cause this state to be mixed with other representations. Jaffe and Wilczek (JW)¹³⁾ proposed the ideal mixing of a pentaquark octet with $\overline{10}$, that is, they predicted the two nucleon states

$$\begin{aligned} N_{ud}^* &= (\bar{u}u + \bar{d}d) \times (uud \text{ or } ddu), \\ N_s^* &= \bar{s}s \times (uud \text{ or } ddu). \end{aligned} \quad (2.2)$$

JW further identified these two states as the two lowest $1/2^+$ resonances, $N(1440)$ and $N(1710)$. It may, however, be possible that they correspond to, for example, the $1/2^-$ states $N(1535)$ and $N(1650)$. If so, it is possible that Θ^+ has negative parity.

Diquarks	J^π	color	flavor
$S_a^5 \equiv \epsilon_{abc}(u_b^T C \gamma^5 d_c)$	0^+	$\mathbf{3}$	$\mathbf{3}$
$\bar{S}_a \equiv \epsilon_{abc}(u_b^T C d_c)$	0^-	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
$\bar{U}_a^5 \equiv \epsilon_{abc}(d_b^T C \gamma^5 s_c)$	0^+	$\mathbf{3}$	$\mathbf{3}$
$\bar{U}_a \equiv \epsilon_{abc}(d_b^T C s_c)$	0^-	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
$\bar{D}_a^5 \equiv \epsilon_{abc}(s_b^T C \gamma^5 u_c)$	0^+	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$
$\bar{D}_a \equiv \epsilon_{abc}(s_b^T C u_c)$	0^-	$\mathbf{3}$	$\mathbf{3}$
Pentaquarks	J^π	flavor	# of s, \bar{s}
$\Theta^+ = \epsilon_{abc} S_a \bar{S}_b^5 C \bar{s}_c^T$	$1/2^+$	$\overline{10}$	1
$\Theta_{NK}^+ = \epsilon_{abc}([u_a^T C \gamma^5 d_b]u_c(\bar{s}_d \gamma^5 d_d) + (u \leftrightarrow d))$	$1/2^-$	$\overline{10}$	1
$N_{ud}^* = \epsilon_{abc} S_a \bar{S}_b^5 C \bar{u}_c^T$	$1/2^+$	$\overline{10} + 8$	0
$N_s^* = \epsilon_{abc} \frac{1}{\sqrt{2}}(\bar{U}_a \bar{S}_b^5 + \bar{S}_a \bar{U}_b^5) C \bar{s}_c^T$	$1/2^+$	$\overline{10} + 8$	2
$N_{\overline{10}}^* = (2/3)N_s^* + (1/3)N_{ud}^*$	$1/2^+$	$\overline{10}$	4/3
$\Sigma_{ud}^* = \epsilon_{abc} \frac{1}{\sqrt{2}}(\bar{U}_a \bar{S}_b^5 + \bar{S}_a \bar{U}_b^5) C \bar{u}_c^T$	$1/2^+$	$\overline{10} + 8$	1
$\Sigma_s^* = \epsilon_{abc} \bar{U}_a \bar{U}_b^5 C \bar{s}_c^T$	$1/2^+$	$\overline{10} + 8$	3
$\Sigma_{\overline{10}}^* = (2/3)\Sigma_s^* + (1/3)\Sigma_{ud}^*$	$1/2^+$	$\overline{10}$	5/3
$\Xi_{3/2} = \epsilon_{abc} \bar{U}_a \bar{U}_b^5 C \bar{u}_c^T$	$1/2^+$	$\overline{10}$	2

Table II. The local QCD operators for diquarks and pentaquarks. The color is denoted by a, b and c . $C = i\gamma^2\gamma^0$ is the charge conjugation matrix. Θ_{NK}^+ is the product of N and K with isospin $I = 0$.

A similar ideal mixing is plausible for Σ ($S = -1$) states, while Ξ ($S = -2$) appear in two multiplets, Ξ^* ($I = 3/2$) in $\overline{10}$ and Ξ^* ($I = 1/2$) in 8 . They do not mix with each other because of the isospin symmetry. The effective number of the strange quarks, n_s , is the key to understanding the $\overline{10}$ structure and its mixing with 8 . In Table II, we list the flavor components of the pentaquark states in the

form of the local QCD operators. We use “diquark” notation, in which two quarks are combined into the color $\bar{3}$ (antisymmetric), the flavor $\bar{3}$ (antisymmetric) and the spin 0 (antisymmetric) states. It is seen from Eq. (2.1) that the average number of strange quarks is 1 for Θ^+ , and it increases to $n_s = 4/3$ for N^* , $n_s = 5/3$ for Σ^* , and $n_s = 2$ for Ξ^* in the $\overline{10}$ representation.

It is easy to see that the maximal breaking of $SU(3)$ leads to the ideal mixing, where $n_s(N_{ud}^*) = 0$, while $n_s(N_s^*) = 2$. Similarly, $n_s(\Sigma_{ud}^*) = 1$ and $n_s(\Sigma_s^*) = 3$. Thus, after the ideal mixing, the spectrum should appear as $N_{ud}, (\Theta^+, \Sigma_{ud}^*), (N_s^*, \Xi_{3/2}^*, \Xi_{1/2}^*) \Sigma_s^*$, where the multiplets in the parentheses have the same value of n_s and are degenerate to leading order in m_s . JW predicted the masses of the $\Xi_{3/2}^*$ and $\Xi_{1/2}^*$ states to be around 1750 MeV, while the original result of DPP predicts a much larger mass for $\Xi_{3/2}^*$.

§3. Theory I: Soliton model

3.1. Skyrmion

The search for the $S = +1$ pentaquark at SPring-8 was motivated by a theoretical prediction of DPP.⁵⁾ That study is based on the Skyrme soliton (Skyrmion) model of the baryon,¹⁴⁾ which utilizes a topological soliton solution in the $SU(3) \times SU(3)$ chiral symmetric nonlinear sigma model. Their model has specific $SU(3)$ breaking terms, indicated by the chiral quark model, but the strengths of the individual terms of the effective Lagrangian are determined phenomenologically.

In the Skyrme model, the classical soliton solution is invariant under neither $SU(3)$ nor spatial rotations. The canonical form is the $SU(2)$ hedgehog imbedded in $U \in SU(3)$:

$$U(\vec{r}) = \exp(i\vec{\lambda} \cdot \hat{r}F(r)), \quad (3.1)$$

where

$$\vec{\lambda} \equiv (\lambda_1, \lambda_2, \lambda_3) = \begin{pmatrix} \vec{\tau} & 0 \\ 0 & 0 \end{pmatrix}.$$

The profile function $F(r)$ represents the radial shape of the soliton and satisfies the boundary conditions $F(r=0) = \pi$ and $F(r \rightarrow \infty) = 0$, so that the topological winding number is equal to 1, which is identified with the baryon number. To recover the symmetries and thus to specify the spin and flavor quantum numbers, we need to “rotate” the Skyrmion in $SU(3)$ flavor space and project the appropriate quantum numbers. This method of quantization is called the ‘rigid rotor quantization’. In the $SU(3)$ Skyrme model with $N_c = 3$, the lowest-lying states are the flavor octet and decuplet states, which correspond to the observed ground state baryons¹⁵⁾ and are also consistent with the quark model prediction with three quarks in the baryon. The next possible representation, $\overline{10}$, is not composed of three quarks and therefore is exotic. Its spin and parity are restricted to $1/2^+$ in this model.

DPP assumed that the known nucleon resonance $N^*(1710)$ is the $Y = 1$ member of the $\overline{10}$ multiplet and derived the (Gell-Mann–Okubo) mass formula

$$M = (1890 - 180Y) \text{ MeV}, \quad (3.2)$$

using the parameters of the Hamiltonian determined so as to reproduce the octet and decuplet baryons. With this formula, the mass of the $Y = 2$ member ($B = 1$ and $S = +1$) is predicted to be $M = 1530$ MeV. This member was first called Z^* but it is now called Θ^+ . They also calculated the width of Θ^+ in the Skyrme model, assuming the simplest meson baryon coupling operator. They found that its width may be as small as 15 MeV.^{*)}

It is important to note that the Skyrme model predicts only positive parity states, unless non-collective meson fluctuations are introduced around the Skyrmion. There is another constraint, namely that the spin is also specified by the $SU(3)$ representation. This constraint is imposed by the Wess-Zumino-Witten term; that is, it results from the stipulation that the allowed representation contains a member with $(I = J, Y = 1)$. Thus, the spin should be equal to the isospin of the $Y = 1$ member of the specified representation.¹⁵⁾ Hence, $J^\pi = 1/2^+$ is the only allowed spin for the $\overline{10}$ flavor state.

There are additional studies based on the Skyrmion model. Their main achievement is to take account of the mixings of other representation due to the $SU(3)$ breaking and to fit the parameter values accordingly. Yabu and Ando¹⁷⁾ carried out a full study of the representation mixing in the octet and decuplet baryons some time ago, and the basic idea is given in their paper. In the case of $\overline{10}$, mixings of 8 and 27 have been introduced, but the qualitative features do not change as a result of the mixings.¹⁸⁾ Diakonov and Petrov¹⁹⁾ have reexamined their study after the discovery of Ξ^* , finding that the choices of the parameter values can be changed so that a smaller mass of $\Xi_{3/2}^*$ is obtained.

We would like to point out here that the Skyrme model in general predicts many (sharp) resonances, which may not necessarily correspond to real baryons, because the model is assumed to represent QCD at large N_c . There is a consensus that there is a significant $1/N_c$ correction, especially for the excited states. Furthermore, several authors have pointed out that the rigid rotor quantization for the $\overline{10}$ representation is not consistent with the large N_c expansion.^{20), 21)}

3.2. K^+ Skyrmion bound state approach

Itzhaki et al.²¹⁾ studied an alternative quantization of the $SU(3)$ Skyrmion, the kaon-Skyrmion bound state approach, and they compared their method with the rigid rotor quantization. This alternative was originally proposed by Callan and Klebanov²²⁾ in the context of describing the strange baryon octet and decuplet. They pointed out that the WZW term is repulsive (attractive) for K^+ (K^-), and the K^+ bound state does not exist in their original work. Itzhaki et al.²¹⁾ reexamined this case and found that if the kaon mass is as large as 1 GeV, then a bound state may appear. In that case, the most probable quantum number is $1/2^+$.

Hosaka²³⁾ pointed out that the chiral bag model exhibits similar qualitative features and predicts a positive parity ground state for Θ^+ . The chiral bag model has a solution of hedgehog form and, in a sense, interpolates between the two extreme

^{*)} Subsequently, it was pointed out that there is a numerical error in their calculation, and the correct value is 30 MeV.¹⁶⁾

pictures, the quark model (MIT bag model) and the Skyrmion model. Hosaka considered a configuration of 4 ud quarks and 1 \bar{s} as a function of the strength of the pion field at the surface of the bag. The first three light quarks occupy the lowest $K = 0$ state. (K is the grandspin defined by $\vec{K} \equiv \vec{I} + \vec{J}$.) The \bar{s} state is almost independent of the pion field. The fourth light quark is found to occupy the negative parity $K^\pi = 1^-$ state if the surrounding pion field is sufficiently strong. This happens because the energy of the $K^\pi = 1^-$ state goes below the energy of the $K^\pi = 1^+$ state for a stronger pion field. In this case, the total spin and isospin are

$$uudd(I = 0, J^\pi = 1^-) + \bar{s}(J^\pi = 1/2^-) \rightarrow J^\pi = 1/2^+ \text{ and } 3/2^+.$$

This mechanism is qualitatively consistent with the kaon-Skyrmion bound state.

The kaon bound state, or the chiral bag model, may be more appropriate than the rigid-rotor quantized Skyrmion. But these two approaches cannot be extended to all the members of $\bar{10}$. Thus we need new ideas to describe the Ξ^* pentaquark.

§4. Theory II: Quark Models

As seen above, the predictions obtained from the Skyrmion model apply only to states with limited quantum numbers. In contrast, QCD with quarks and gluons should have a richer spectrum with various spin and flavor quantum numbers. We therefore consider the general quark model in this section and find what it can yield for the pentaquark states.

4.1. Symmetry in quark models

First we consider the symmetry restrictions on the tetra-quark states composed of u^2d^2 quarks. We consider only the $I = 0$ and $C = 3$ states, so that they are a part of the Θ^+ baryon. With four quarks, this state possesses the following symmetry:*)

$$I = 0 \quad \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \otimes C = 3 \quad \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \square & \\ \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \square & \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \square & & \\ \hline \end{array}$$

The remaining degrees of freedom corresponds to the spin and orbital motion. For simplicity, we consider the classification in the nonrelativistic limit. If we assume that the ground state is the orbitally symmetric state with $L = 0$, then the only allowed spin is $S = 1$ with $[31]$ symmetry.

However, the conventional quark model yields the conclusion that the spin dependent interactions, especially the hyperfine interaction (HF), are important. The HF favors lower spin states, and therefore the $S = 1$ tetraquark state is less favored. On the other hand, the $S = 0$ state is allowed only with the mixed orbital symmetry, $[31]$, having $L = 1$. Therefore the combined total angular momentum of the tetraquark is $j = 1$. Such a state is favored by the HF interaction, but the cost is its greater kinetic energy.

*) We here consider the case for Θ^+ , but the same symmetry classification applies to other pentaquark states. For the other states, we replace $I = 0$ by the flavor representation, $F = \bar{6}$.

Thus, the possible low energy configurations for the Θ^+ baryon are

- (A) $u^2 d^2 (L = 0, S = 1, j^\pi = 1^+) + \bar{s}(1/2^-) : J^\pi = 1/2^- \text{ or } 3/2^-,$
- (B) $u^2 d^2 (L = 1, S = 0, j^\pi = 1^-) + \bar{s}(1/2^-) : J^\pi = 1/2^+ \text{ or } 3/2^+.$

In either case, there should exist a spin partner, $J = 3/2$ state, associated with the “ground” $J = 1/2$ state.^{*)} However, the origin of the $J = 1/2 - 3/2$ splitting differs. For case (A), the spin-spin interaction, $\propto \vec{S}(u^2 d^2) \cdot \vec{S}(\bar{s})$, is responsible for splitting them, while for case (B), the (multibody) spin-orbit force, $\propto \vec{L}(u^2 d^2) \cdot \vec{S}(\bar{s})$, is the interaction which causes the splitting. In the ordinary hadron spectrum, the spin-spin splitting is much larger than the spin-orbit splitting. Furthermore the “three-body” LS force in case (B) is expected to be smaller than the two-body LS. According to Dudek and Close,²⁴⁾ the splitting in case (B) is of the order of a few tens of MeV, while the spin-spin splitting in case (A) is ~ 200 MeV. In any case, the observation of the $J = 3/2$ partner of Θ^+ would be critical in order to distinguish among the models of the structure of the pentaquark baryons.

We consider the above two cases further in order to determine which of the two is preferable in the constituent quark model approaches. The main question is which of the following two competing energies is larger:

- Orbital excitation energy: $\Delta E(\Delta L = 1) = E(L = 1) - E(L = 0)$,
- Hyperfine (HF) interaction: $\Delta E(\Delta S = 1) = E(S = 1) - E(S = 0)$.

If $\Delta E(\Delta L = 1) > \Delta E(\Delta S = 1)$, case (A) with $J^\pi = 1/2^-$ may give the ground state, while if $\Delta E(\Delta L = 1) < \Delta E(\Delta S = 1)$, case (B) with the $1/2^+$ ground state may be realized.

4.2. Constituent quark model

A simple constituent quark model is employed here to estimate the above two quantities using the knowledge we have regarding the ordinary hadron spectrum. The constituent quark is known to be a very useful concept in describing the meson and baryon spectra and their properties. It has a mass of order 300 MeV, which is believed to be generated by the chiral symmetry breaking in QCD. It can be derived by solving the Dyson-Schwinger equation for the quark propagator.²⁵⁾ The interaction kernel has strong enhancement in the low momentum region, and it induces dynamical chiral symmetry breaking and generates a momentum dependent effective mass. Thus the quark acquires the self-energy and is renormalized. It is important to note that the conserved currents, such as the isospin, I , the hypercharge, Y , and color charge, C , are not renormalized.

Therefore, the constituent quark has the same quantum numbers as the QCD quark but its mass is given roughly by $M_B/3$. We choose, as typical values of the quark mass, $m_{u,d} \sim 360$ MeV and $m_s \sim 540$ MeV. Then the sums of the quark

^{*)} Such a state with the same flavor symmetry does not necessarily exist in the Skyrmin picture.

masses for the baryons become

$$\sum m_q = \begin{cases} 1080 \text{ MeV for } N \text{ and } \Delta, \\ 1260 \text{ MeV for } \Lambda, \Sigma \text{ and } \Sigma^*, \\ 1980 \text{ MeV for } u^2 d^2 \bar{s}. \end{cases} \quad (4.1)$$

Assuming that the residual interactions are weak,^{*)} these values provide estimates of the average masses of the indicated baryons.

4.3. Orbital excitation

We first estimate the orbital excitation energy, that is, the energy necessary for the excitation of a constituent quark from the lowest single particle energy level to the first excited level with $L = 1$. This energy can be estimated from the excitation energy of the negative-parity baryon resonance, e.g., $N^*(1535) - N(940)$, $\Delta E \sim 600$ MeV. This is, however, an overestimate, because it contains the difference of the hyperfine interactions in the two baryons.

A more reliable estimate may be obtained by calculating the difference of the kinetic energies in the quark-diquark system. For quarks of constituent mass ~ 360 MeV, the reduced mass becomes

$$\mu = 360 \times \frac{3}{4} = 270 \text{ MeV}. \quad (4.2)$$

Assuming that the distance between the quark and the diquark is about 0.6 fm, which is a typical size of the quark wave function in the hadron, we obtain $\Delta E(\Delta L = 1) \sim 400$ MeV.

It is interesting that the JW diquark model may require even a larger kinetic energy excess, because, if we assume that the diquark mass is about 420 MeV, then the reduced mass for the diquarks is 210 MeV.

4.4. Hyperfine interaction

The importance of the hyperfine interaction in the hadron spectrum was first pointed out by DeRujula, Georgi and Glashow²⁶⁾ in the context of one-gluon exchange. It is the spin-spin interaction given by the magnetic component of the one-gluon exchange (Fig. 2(a)) and is written in nonrelativistic form as

$$\frac{\alpha_s}{m_i m_j} (\lambda_i \cdot \lambda_j) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij}). \quad (4.3)$$

This is known as the color magnetic (CM) interaction. Its main contribution in the baryon spectrum is to split the octet and decuplet baryons, such as N and Δ .

We simplify the form of the CM as

$$\Sigma_{\text{CM}} = \sum_{i < j} \Delta_{\text{CM}} \xi_{ij} (\lambda_i \cdot \lambda_j) (\sigma_i \cdot \sigma_j) \quad (4.4)$$

^{*)} A confinement force is, of course, required to combine quarks into bound states. This effect is assumed to be within the constituent quark mass; i.e., m_q is regarded as the single particle energy of the quarks in the ground state.

and fix the strength Δ_{CM} using the $N - \Delta$ mass difference, which leads to

$$\Delta_{\text{CM}} = 18.75 \text{ MeV}. \quad (4.5)$$

Here ξ denotes a factor due to the $SU(3)$ symmetry breaking, defined by

$$\xi_{ij} = \begin{cases} 1 & \text{for } (ij) = (uu), (ud) \text{ or } (dd), \\ m_u/m_s & \text{for } (ij) = (us) \text{ or } (ds), \\ (m_u/m_s)^2 & \text{for } (ij) = (ss). \end{cases} \quad (4.6)$$

This is introduced so that the mass splitting ($\sim 80 \text{ MeV}$) of Λ and Σ is reproduced by the CM interaction if we take $m_s/m_u \sim 5/3$.

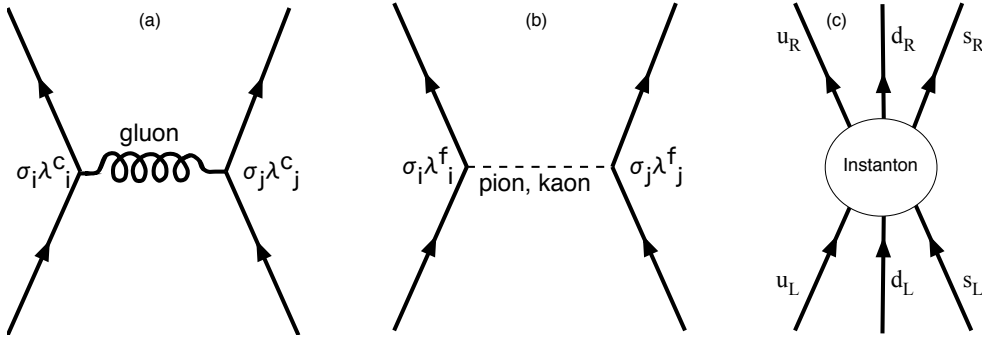


Fig. 2. (a) CM from the one gluon exchange, (b) FD from the exchange of the pseudoscalar mesons and (c) III due to the quark-instanton coupling.

Other possible origins of the HF interaction have been discussed in the literature. One of the approaches predicts pseudoscalar meson exchange interactions between quarks inside the baryon (Fig. 2(b)). We call this interaction the “flavor dependent” (FD) HF interaction.²⁷⁾ This force depends on the flavor and spin of the quarks and is given in simplified form as

$$\Sigma_{\text{FD}} = -\Delta_{\text{FD}} \sum_{i < j} \xi_{ij} (\lambda_i^f \cdot \lambda_j^f) (\vec{\sigma}_i \cdot \vec{\sigma}_j) \quad (4.7)$$

The strength can be determined again using the $N - \Delta$ mass difference: $\Delta_{\text{FD}} \sim 30 \text{ MeV}$.

The third possible origin of the HF interaction is the instanton induced interaction (III) (Fig. 2(c)). It was shown by Shuryak and Rosner²⁸⁾ and Takeuchi and Oka²⁹⁾ that the III gives the correct HF interaction for the ground-state baryon spectrum. Takeuchi³⁰⁾ further showed that the spin-orbit interaction in the P-wave baryons is also described well by the III. The III is also known as the Kobayashi-Maskawa-’t Hooft (KMT) interaction, and it results from the coupling of light quarks with the instanton. It is given as follows:

$$V_{\text{III}}^{(3)} = V^{(3)} \sum_{(ijk)} \mathcal{A}^f \left[1 - \frac{1}{7} (\vec{\sigma}_i \cdot \vec{\sigma}_j + \vec{\sigma}_j \cdot \vec{\sigma}_k + \vec{\sigma}_k \cdot \vec{\sigma}_i) \right] \delta(\vec{r}_{ij}) \delta(\vec{r}_{jk}),$$

$$V_{\text{III}}^{(2)} = V^{(2)} \sum_{i < j} \mathcal{A}^f \left[1 - \frac{1}{5} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right] \delta(\vec{r}_{ij}). \quad (4.8)$$

It contains the flavor antisymmetric ($u - d - s$) 3-body repulsion and also the flavor antisymmetric 2-body attraction, which is induced by contracting the 3-body force by contracting a pair consisting of a quark and an antiquark.

The two-body part of the III can be simplified into the HF form,

$$\Sigma_{\text{III}} = \Delta_{\text{III}} \sum_{i < j} \mathcal{A}_{ij}^f \xi_{ij} \left[1 - \frac{1}{5} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right], \quad (4.9)$$

and its strength is given by $\Delta_{\text{III}} \sim 125$ MeV.

Although the III is not uniquely identified in the baryon spectrum, its importance is known in the meson spectrum and the dibaryon spectrum. 't Hooft³¹⁾ pointed out that the III breaks $U_A(1)$ symmetry and thus gives the η and η' mass difference. Indeed, the above form of the III was used by Hatsuda and Kunihiro³²⁾ in an NJL type model to reproduce the pseudoscalar meson spectrum. The importance of the three-body III in the H dibaryon prediction was pointed out by Takeuchi and Oka;³⁴⁾ that is, the three-body repulsion pushes the H mass above the 2Λ threshold.

Which of the contributions of the HF is most important? It is quite likely that the actual HF interaction is a combination of Σ_{CM} , Σ_{FD} and Σ_{III} . But, we would like to stress that the axial $U(1)$ symmetry breaking caused by the III manifests itself in the pseudoscalar mesons, η and η' . Thus at least a part of the HF interaction must come from the III. This fact is also favorable, because the CM interaction can be reduced so that an abnormally large α_s is not necessary to explain the baryon spectrum.

4.5. Θ^+ mass

Now, using the above simplified forms of the HF interactions, we estimate the baryon masses and apply the same calculation to the pentaquarks. The masses of the nucleon and the Δ resonance are given in the CM model by

$$M_N = 3m_q + \langle \Sigma_{\text{CM}} \rangle_N = 360 \times 3 - 150 = 930 \text{ MeV}, \quad (4.10)$$

$$M_\Delta = 3m_q + \langle \Sigma_{\text{CM}} \rangle_\Delta = 360 \times 3 + 150 = 1230 \text{ MeV}. \quad (4.11)$$

Similarly, the $SU(3)$ breaking effects are properly taken into account in that model, the Λ mass is given by

$$M_\Lambda = 3m_q + \Delta m + \langle \Sigma_{\text{CM}} \rangle_\Lambda = 360 \times 3 + 180 - 150 = 1110 \text{ MeV}, \quad (4.12)$$

where the mass difference between the s quark and the u and d quarks is given by

$$\Delta m = m_s - m_q \sim 540 - 360 = 180 \text{ MeV}. \quad (4.13)$$

We note again that all the spin dependent interactions are assumed to be included in the “single particle” energy, i.e., the constituent quark mass.

It is found that the CM is not sufficient for the H dibaryon, a system of six quarks, represented by $u^2d^2s^2$, with strangeness -2 . The CM predicts a deep bound state for H , whose mass is estimated as

$$M_H = 6m_q + 2\Delta m + \langle \Sigma_{\text{CM}} \rangle_H = 360 \times 6 + 2 \times 180 - 450 = 2070 \text{ MeV}, \quad (4.14)$$

while the $\Lambda\Lambda$ threshold is 2231 MeV. Since the prediction of the bound H by Jaffe³³⁾ in 1977, experimental searches were performed for twenty years with a negative result. This is the case in which the 3-body III gives approximately a 160 MeV repulsion.³⁴⁾ Although the other baryons are reproduced fairly well by any of the three models, the fact that the H dibaryon has not been observed gives reason to believe that the instanton is necessary.

Now we evaluate the mass of the Θ^+ using the simple model described above. For the negative parity Θ^+ ($1/2^-$), we obtain

$$M_{\Theta^+} = 5m_q + \Delta m + \langle \Sigma_{\text{CM}} \rangle_{\Theta^+} = 360 \times 5 + 180 - 250 \sim 1730 \text{ MeV}. \quad (4.15)$$

This estimate may look too crude because it ignores, for instance, the difference between the confinement of the 3-quark baryon and the pentaquarks, and also the other spin independent forces. It has been shown, however, that a more realistic constituent quark model, such as the MIT bag model, gives similar values for the pentaquark masses.³⁵⁾ Thus the CM model requires an extra attraction of about 200 MeV for Θ^+ ($1/2^-$).

The effect of FD on Θ^+ has been studied by Stancu and Riska³⁶⁾ and also by Jennings and Maltman³⁷⁾ and Bijker et al.³⁸⁾ The latter two works also compare FD with CM. It is found there that the FD interaction is slightly less repulsive.

The instanton plays a crucial role also in Θ^+ . The III (2-body) is strongly attractive, while the III (3-body) is slightly repulsive. We find the Θ^+ mass employing the III to be the surprisingly small value

$$M_{\Theta^+} = 5m_q + \Delta m + \langle \Sigma_{\text{III}} \rangle_{\Theta^+} = 360 \times 5 + 180 - 510 + 50 \sim 1520 \text{ MeV}, \quad (4.16)$$

where the large attraction (-510 MeV) comes from the 2-body III, and the last term, the 50 MeV repulsion, is the contribution of the 3-body III. Although this result is very close to the observed Θ^+ mass, this estimate may not be fully realistic. This is because the strong attraction is estimated perturbatively, assuming that the quark wave functions of the 3-quark baryon and the pentaquarks are the same. A realistic estimate employing the MIT bag model³⁹⁾ has been carried out and shows that the Θ^+ mass with the full III is about 1600 MeV.

In contrast to the situation described above, the positive parity baryon has a problem that it typically gives a mass that is 100 MeV larger than that of the negative parity state. In the CM case, we have

$$\begin{aligned} M_{\Theta^+} &= 5m_q + \Delta m + \langle \Sigma_{\text{CM}} \rangle_{\Theta^+} + \Delta E(\Delta L = 1) \\ &= 360 \times 5 + 180 - 620 + 450 \sim 1810 \text{ MeV}. \end{aligned} \quad (4.17)$$

The attraction due to the HF interaction is indeed strong, but it is cancelled by the kinetic energy of the $L = 1$ orbital motion. The other models yield the same

conclusion for $1/2^+$ as long as quark configurations based on the independent particle model, or the quark shell model, are considered.

The above exercises show that (1) the constituent quark model for the $1/2^-$ pentaquark tends to yield a larger mass than the observed Θ^+ , though (2) the instanton induced interaction is a promising source of strong attraction for the pentaquark and (3) the $1/2^+$ state may not realize a lower energy than the $1/2^-$ state in the independent-particle quark models.

There are many model calculations that claim a positive parity state of lower energy than $1/2^-$. It is, however, fair to say that the majority of such calculations contain an arbitrary parameter that is fitted to the observed Θ^+ mass or estimates the kinetic energy associated with $L = 1$ unreasonably small. One such example is the estimate in Ref.,^{37),40)} which employ $\Delta E(\Delta L = 1) = 210 - 250$ MeV, based on the splitting of the $N^*(1440)$ from the $N(940)$. But it is clearly unjustified to assume that the $N^*(1440)$ is a $2\hbar\omega$ excited state and $\Delta E(\Delta L = 1) = \hbar\omega$.

One possibility is to deviate from the independent particle picture. Recently, a dynamical (but semi-classical) five-quark calculation⁴¹⁾ was carried out with a model Hamiltonian that contains a linear confinement potential plus the CM interaction. That calculation employs the antisymmetrized molecular dynamics (AMD) method, and the result shows that the $J^\pi = (1/2^+, 1/2^-)I = 0$ and $J^\pi = (3/2^-)I = 1$ states are almost degenerate as the lowest energy states. The authors claim that the diquark-type correlation causes the $1/2^+$ state to have a smaller mass. This is the subject of the next section.

4.6. Diquarks

In order to decrease the masses of the pentaquark states to the observed values in the quark model, Jaffe and Wilczek (JW)¹³⁾ proposed the diquark model. A similar model was also proposed by Karliner and Lipkin (KL).⁴⁰⁾

The diquark is a strongly correlated quark pair. In QCD, both the gluon exchange interaction and the instanton induced interaction favor the spin-singlet and color-antisymmetric diquark combination,

$$[ud]_0 \quad I = 0 \quad C = \bar{3} \quad J^\pi = 0^+. \quad (4-18)$$

The instanton model calculation predicts a $[ud]_0$ diquark mass as small as 420 MeV.

In the JW model, the structure of the Θ^+ is considered in terms of two diquarks and a strange antiquark system. It is important that the color-singlet bound state of these three objects requires an orbital excitation, either to $L = 1$ or to a state with a nodal radial wave function. This can be understood from symmetry considerations: The two $[ud]_0$ should be antisymmetric in color so that a total color singlet state is obtained, while they should be symmetric overall because they are “bosons.” Thus the relative motion between the diquarks should have $L = 1$. The mass of the Θ^+ is estimated as

$$M \sim 2M_D + m_s + \Delta E(\Delta L) \sim 840 + 560 + 450 = 1850 \text{ MeV}. \quad (4-19)$$

In the KL model, the combination of a diquark and a triquark $ud\bar{s}$ is considered,

and the Θ^+ mass is estimated. It is also assumed that the relative angular momentum is $L = 1$, but the estimated value of $\Delta E(\Delta L)$ is much smaller, ~ 209 MeV.

How realistic are the diquarks? An attempt was made in lattice QCD to calculate the diquark mass. As the diquark is not color singlet, it is necessary to fix the gauge to calculate the mass. Wetzorke and Karsch⁴²⁾ used the Landau gauge and derived the spectrum of the diquarks. A recent calculation employing the maximal entropy method for the diquark spectral function yields the conclusion that the mass of the $[ud]_0$ diquark is about 600 MeV. There is also an approach using the Bethe-Salpeter equation within rainbow-ladder QCD⁴³⁾ that predicts a diquark mass ~ 800 MeV.

We point out that the diquark picture for the pentaquark system requires reexamination of the nucleon and other ground state baryons with the same footing. An interesting quantity is the magnetic moments of the ground state baryons. Because the diquark enhancement may break the $SU(6)$ symmetry, the relations among the magnetic moments of the octet baryons may not hold. For instance, the ratio of the magnetic moments of the proton and neutron is given in the diquark limit by

$$\mu_p/\mu_n = e_u/e_d = -2 \quad (4.20)$$

if the quarks do not have anomalous moments.

In summary, the situation is not at all clear in the “constituent” quark model. In fact, the quark model has been tested in the case of the nonexotic mesons and baryons, but it may be necessary to modify it to accommodate the pentaquarks and other exotic states. The suggested strong correlation realized in the diquarks, if it is real, may make it necessary to reconsider the basic dynamics of the constituent quark model. The width is another quantity that is to be explained in the quark model. The “fall-apart” theory is conventionally employed and predicts a width of the pentaquarks that is too large, but the validity of such a theory has not yet been confirmed.

§5. QCD prediction

In the situation that the quark model cannot provide definite predictions for the spin, parity and structure of the pentaquark state, we desperately need a QCD calculation. Several attempts have been made to apply QCD directly to this problem.

5.1. QCD sum rules

The QCD sum rule⁴⁴⁾ has been applied to the Θ^+ spectrum by Zhu,⁴⁵⁾ Matheus et al.,⁴⁶⁾ and Sugiyama et al. (SDO).⁴⁷⁾ The first two approaches consider only the chiral even terms of the sum rule and do not consider the parity projection.

SDO made the first attempt to determine the parity and mass of Θ^+ from the QCD sum rule, assuming that its spin is $\frac{1}{2}$. The QCD sum rule is derived by computing the relevant correlation function in two ways, with the operator product expansion (OPE) in the deeply Euclidean region, and with a phenomenological parameterization of the spectral function. This sum rule relates hadron properties directly to the QCD vacuum condensates, such as $\langle \bar{q}q \rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, as well as to the

other fundamental constants, such as m_s .

In Table II, possible local operators for the pentaquark systems are listed. We employ the following interpolating field operator for Θ^+ :

$$J(x) = \epsilon_{abc} \bar{S}_a S_b^5 C \bar{s}_g^T(x). \quad (5.1)$$

Here, a, b, c, \dots are color indices and $C = i\gamma^2\gamma^0$. Also, $S_c^5(x) = \epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)$ and $S_c(x) = \epsilon_{abc} u_a^T(x) C d_b(x)$ are the scalar (0^+) and the pseudoscalar (0^-) ud diquark operators. They both belong to color $\bar{3}$ and $I = 0$. The operator $J(x)$ produces a baryon with $J = \frac{1}{2}$, $I = 0$ and strangeness $+1$. One of the advantages of $J(x)$ is that its coupling to the main continuum state, NK , is believed to be small, because it cannot be decomposed into a product of $N(3q)$ and $K(q\bar{q})$ operators in the nonrelativistic limit.

It is important that the correlation function of $J(x)$ contains both positive and negative parity components. Therefore it is necessary for the parity projection to determine the parity of the pentaquark state. Such a technique was developed by Jido et al.⁴⁸⁾ and it has been applied to the baryon excited states. Here, we follow the same procedure for the pentaquark correlation function.

We consider the retarded Green's function and choose the rest frame, $\vec{q} = 0$,

$$\Pi(q_0) = \int d^4x e^{iq \cdot x} i \langle 0 | \theta(x^0) J(x) \bar{J}(0) | 0 \rangle |_{\vec{q}=0}. \quad (5.2)$$

Then the imaginary part of Π contains two functions, which are the sum and difference of the positive and negative parity spectral functions, ρ^\pm ,

$$\begin{aligned} \frac{1}{\pi} \text{Im} \Pi(q_0) &= A(q_0) \gamma^0 + B(q_0) \\ A(q_0) &= \frac{1}{2} (\rho^+(q_0) + \rho^-(q_0)) \\ B(q_0) &= \frac{1}{2} (\rho^+(q_0) - \rho^-(q_0)), \end{aligned} \quad (5.3)$$

or equivalently,

$$\rho^\pm(q_0) = A(q_0) \pm B(q_0). \quad (5.4)$$

It happens that the part B in the OPE contains only the chiral-odd terms, such as $\langle \bar{q}q \rangle$ and $\langle \bar{q}g_s \sigma \cdot Gq \rangle$, and thus the chiral symmetry breaking is responsible for the parity splitting.

The sum rule is obtained by comparing the OPE of the correlation function and explicit forms of the spectral functions using analytic continuation. The spectral function is commonly parametrized as a pole plus the continuum contribution:

$$\rho_{\text{Phe}}^\pm(q_0) = |\lambda_\pm|^2 \delta(q_0 - m_\pm) + \theta(q_0 - \sqrt{s_{\text{th}}}) \rho_{\text{CONT}}^\pm(q_0). \quad (5.5)$$

Here, the continuum part is assumed to be identical to the corresponding OPE function above the threshold $\sqrt{s_{\text{th}}}$. In order to both enhance the pole part and also

suppress the higher-dimensional terms of the OPE, we introduce a weight function of the “Borel” type. Then the sum rule is obtained as

$$\int dq_0 W(q_0) \rho_{\text{Phen}}^{\pm}(q_0) = \int dq_0 W(q_0) \rho_{\text{OPE}}^{\pm}(q_0). \quad (5.6)$$

with $W(q_0) = \exp\left(-\frac{q_0^2}{M^2}\right)$, where M is the relevant (Borel) mass scale for the baryon. The values of the QCD parameters, m_s , $\langle \bar{s}s \rangle$, $\langle \bar{s}g_s \sigma \cdot Gs \rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, are chosen so that the strange baryon masses are reproduced in the sum rule. The values we choose are as follows: $m_s = 0.12$ GeV, $\langle \bar{s}s \rangle = 0.8 \times (-0.23 \text{ GeV})^3$, $m_0^2 \equiv \langle \bar{s}g_s \sigma \cdot Gs \rangle / \langle \bar{s}s \rangle = 0.8 \text{ GeV}^2$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.33 \text{ GeV})^4$.

The residue of the pole, $|\lambda_{\pm}|^2$, in Eq. (5.5) represents the strength with which the IF couples to the physical state, and it should be positive only when the pole is real. We use this condition to determine the parity of the pentaquark state. In Fig. 3, we plot the OPE side (as a function of M) corresponding to $|\lambda_{\pm}|^2 \exp(-m^2/M^2)$. We find that the dimension-5 condensate, $\langle \bar{s}g_s \sigma \cdot Gs \rangle$, gives a large negative contribution to $|\lambda_{+}|^2$, which ends up with a nearly zero or even slightly negative value. This suggests that the pole in the positive-parity spectral function is spurious. In contrast, the large $\langle \bar{s}g_s \sigma \cdot Gs \rangle$ contribution makes $|\lambda_{-}|^2$ positive, and we thus conclude that the obtained negative-parity state is a real state.

The mass of the negative parity Θ^+ is estimated using the sum rule, and we find that the M dependence is rather weak. Therefore the reliability of the sum rule seems good. It is, however, sensitive to the choice of the threshold, and the result is ambiguous to some extent. We have only confirmed that the result is consistent with the observed value.

Summarizing this part, the QCD sum rule shows that (1) the parity splitting is caused by chiral symmetry breaking due to m_s , $\langle \bar{q}q \rangle$ and $\langle \bar{q}g_s \sigma \cdot Gq \rangle$ condensates. (2) The parity of Θ^+ is determined by the positivity of the correlation function and the negative parity state is predicted. (3) The mass of the $1/2^- \Theta^+$ baryon is obtained as $1.3 - 1.7$ GeV.

An important issue regarding the QCD sum rule is the choice of the interpolating field operator. If the IF is inappropriate, the sum rule does not contain the pentaquark contribution, but the KN continuum state may dominate. A recent study⁴⁹⁾ indicates that the continuum effect on the sum rule might change the conclusion concerning the parity assignment. Further studies on the choice of the IF and the sum rule approach are necessary.

5.2. Lattice QCD

Several lattice QCD calculations suggest that the positive parity Θ^+ is very massive. Csikor et al.,⁵⁰⁾ Sasaki⁵¹⁾ and Lee et al.⁵²⁾ calculated the Θ^+ correlation function in quenched lattice QCD. They employed different local interpolating field (IF) operators that do not contain derivatives. They are independent, but from the Fierz transform, it is seen that they are not orthogonal.

Lee et al. have used the IF operator given by the product of N and K , i.e., Θ_{NK}^+ in Table II. They concluded that the correlator is consistent with the NK background and exhibit a resonance state in neither $1/2^-$ nor $1/2^+$. This result

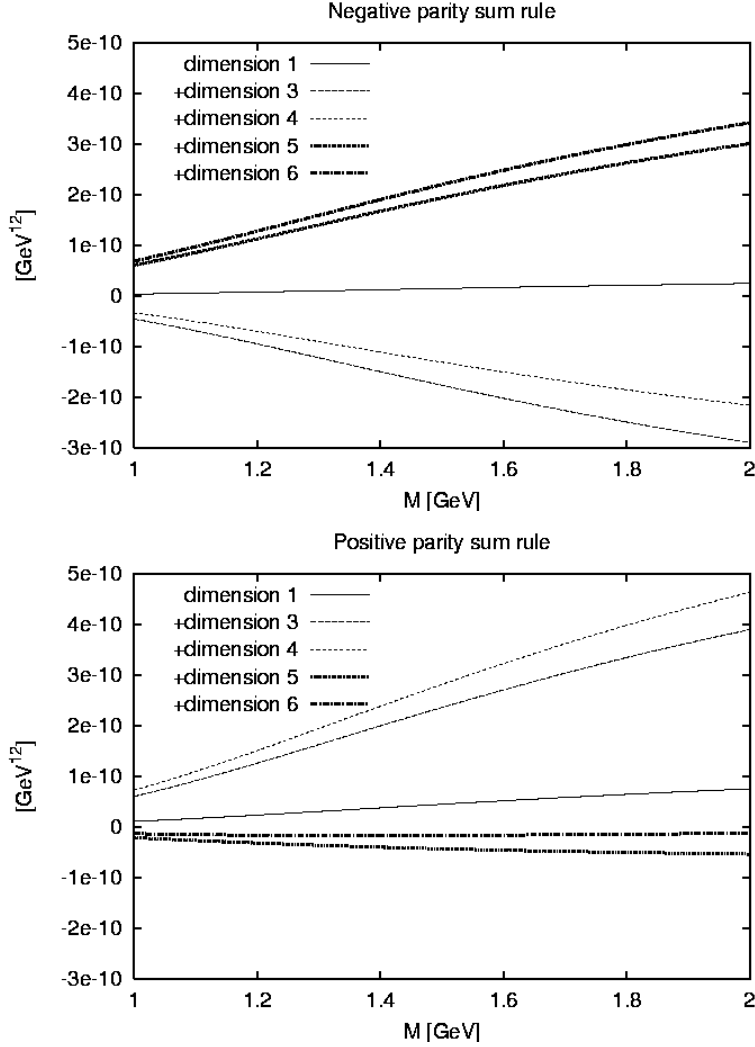


Fig. 3. Contributions from the terms of each dimension added up successively for the negative-parity and positive-parity sum rules with $\sqrt{s_{\text{th}}} = 1.8$ GeV.

can be understood naturally, because their IF has a large overlap with the NK continuum state.

In contrast, in both Refs.⁵⁰⁾ and⁵¹⁾ it is claimed that there exists a resonance state of $1/2^-$ at a mass consistent with the observed Θ^+ . Sasaki employed the same IF as that used in the QCD sum rule in the previous section, and he found that the effective mass extracted from the time dependences of the correlation function possesses two plateaus. One corresponds to the NK threshold and the other indicates a narrow resonance above the NK threshold. It is important to confirm whether such a double plateau structure is not an artifact of lattice QCD.

The Hungarian group⁵⁰⁾ used another IF and found that Θ^+ has $J^\pi = 1/2^-$ and that its mass is consistent with the NK threshold. It is not clear whether they can

claim a resonance above the threshold or not.

It is fair to say that the majority of the calculations agree that the ground state is characterized by $J = 1/2$, $I = 0$, and negative parity, while the positive parity state has a mass that is greater at least by a few hundred MeV. These calculations also show that the results obtained to this time do not depend strongly on the choice of the interpolating field.

§6. Conclusion

The conclusions of this article are itemized in the following.

- QCD predicts the existence of a Θ^+ ($J^\pi = 1/2^+$, $I = 0$), although it may not be easy to distinguish Θ^+ from the KN threshold in the sum rule or in lattice QCD.
- Many models suggest positive parity baryons. The soliton models provide a parametrization of the masses of the “pentaquark” states that is consistent with the observed masses. The quark models for the positive parity states seem to often use a kinetic excitation energy that is too small, $\Delta M = M(L = 1) - M(L = 0) \sim 200$ MeV, while a reasonable estimate is about 400 – 450 MeV. Unless an exotic idea, such as a strong diquark correlation, is introduced, it seems unlikely that the $1/2^-$ state will have a higher energy than the $1/2^+$ state in the quark model.
- The quark model predicts a mass of Θ^+ that is larger than the observed value by about 100 MeV - 200 MeV. The discrepancy is larger for $1/2^+$. The idea of the strong diquark correlation may require confirmation from the QCD viewpoint.
- The real challenge is to account for the small width. It was even pointed out that consistency with the existing KN phase shift analyses necessitates a tiny width, as small as 0.1 MeV, in particular for the negative parity Θ^+ .⁵³⁾ A mechanism yielding strong suppression of the decay is desperately needed.

New experiments with better statistics and with different production mechanisms have already appeared and will continue to appear. Most theoretical models predict not one but in fact many pentaquark states, and therefore observations of other states are extremely important. Among them, the spin $J = 3/2$ partner and the other members of the antidecuplet, as well as the octet, which may be mixed with $\overline{10}$, are particularly anticipated.

Recently, new theoretical papers on the pentaquark appear daily. We need further study of the width and the production mechanisms as well as realistic (not toy-model) calculations based on QCD. Lattice QCD with improved statistics and other interpolating field operators is also necessary to confirm the nature of pentaquarks.

It would also be fascinating to determine whether pentaquark spectroscopy is related to other newly-observed narrow hadron states, such as D_s^* and $X(3872)$. It would be interesting to study whether the heavy quark plays a key role in these new resonances.

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